Effects of Frictional Force, Flow Rate and Shear Stress on Elliptic Plate using Couple Stress Fluid

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Abstract

The theoretical analysis has been made in this paper to analyze the effects of frictional force, flow rate and shear stress of elliptic plates using couple stress fluid. The modified Reynolds equations are solved using relevant boundary conditions to get the expressions for pressure, frictional force, flow rate and the shear stress. The results are represented graphically for various parameters and it is observed that the effects of shear stress decreases with increase of film thickness both in upper plate and lower plate and frictional force increases with increase in viscosity coefficient and decreases for the increasing values of couple stress parameter. The flow rate decreases with the increasing values of couple stress parameter and the aspect ratio

Keywords: Porous matrix, Couple stress fluid, frictional force, shear stress, flow rate, squeeze film, elliptic plate

1. Introduction

To understand various physical problems, the study of a couple stress fluids is very useful because it consists of a method to express rheologically complex fluids such as liquid crystals, liquids containing long-chain molecules, human and animal blood lubrication. The Stokes [1] couple stress model is the overview of classical theory that allows for polar effects such as the occurrence of couple stresses and body couples. This hypothesis has been used by G.Ramaniah [2] and studied the squeeze films between finite plates lubricated by couple stress fluid and concluded that the squeeze time raises if couple stress is used as the lubricant.

N.B Naduvinamani and A.Siddagounda [3] have calculated the effect of couple stress between circular stepped plates and revealed that the effects of couple stresses improves the pressure of the squeeze film, load sustaining capability, and decreases the reaction time as compared to conventional Newtonian case. The load sustaining capacity decreases by increasing step height. N.B Naduvinamani
et al [4] have taken long porous journal bearings with the couple stress fluids and established that the couple stresses effect increases the load sustaining capacity and thereby extend the squeeze film time compared to Newtonian case. The permeability effect helps to reduce the load sustaining capacity and decrease the squeeze film time as compared to the equivalent solid case.

Shear stress and the normal stress which are arises from the component of force vector are parallel and perpendicular to the cross section of the material respectively. The shear stress on different bearing system was studied by several researchers. Bing-He Ma et.al [5] had studied the accurate measurement of wall shear stress with the elliptic plate and concluded that the wall shear stress obtained and theoretical results are well granted with each others. The viscous shear effects on the squeeze film characteristics predicted by Lin JR [6] and noted that the influences of viscous shear stresses on squeeze film behavior are considerable and enhances the load sustaining capability and thereby increase the reaction time of squeeze film.

Based upon Brinkman model, Jaw-Ren Lin et al [7] has investigated and provided the better result than the other model in enhancing the load carrying capacity and lengthen the response time. Mohamed Nabhani and et al [8] studied the effect of viscosity on finite porous elastic journal bearing lubricated with non-Newtonian couple stress fluid and establish that viscous shearing forces increase the load sustaining capability and friction aspect. Sundarammal et al [9-11] have investigated the squeeze film characteristics between porous elliptic plates with velocity and frictional effects.

2. Experimental Methods

Fig (1) indicates the geometry of the bearing system. Given two elliptical plates each of semi-major axis ‘a’ and semi-minor axis ‘b’, the lower plate is fixed and the upper plate has a porous facing which is supported by a solid wall and it moves towards the lower plate with normal velocity and the lubricant is assumed to be incompressible stokes couple stress fluid and the body forces and body couples are insignificant.

**Figure 1:** The geometry of parallel elliptic plate
Under the usual assumptions, the basic governing equation of hydrodynamic lubrication driven by Stokes takes the form

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]  

(1)

\[
\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^2 u}{\partial z^3}
\]

(2)

\[
\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} - \eta \frac{\partial^2 v}{\partial z^3}
\]

(3)

\[
\frac{\partial p}{\partial z} = 0
\]

(4)

The velocity components along \(x, y, z\) directions are given by \(u, v, w\) respectively, \(p\) denotes the pressure in film region, material constant of dimension of viscosity is given by \(\mu\) and material constant responsible for couple stress is represented by \(\eta\).

The appropriate B.C's are

(i) At \(z = h\)

\[w = V = \frac{d h}{d t}\]  

(5a)

\[u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0\]  

(5b)

(ii) At \(z = 0\)

\[u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0\]  

(6a)

\[w = 0\]  

(6b)

Solving (2) and (3) with the B.C's 5(a) and 5(b) we get

\[
u = \frac{\partial}{\partial x} \left( \frac{z^2 - 2h}{2l^2} + 2l^2 \left[ 1 - \frac{\cosh \left( \frac{2z - h}{z l} \right)}{\cosh \left( \frac{h}{2l} \right)} \right] \right)
\]

(7)

Similarly

\[
u = \frac{\partial}{\partial y} \left( \frac{z^2 - 2h}{2l^2} + 2l^2 \left[ 1 - \frac{\cosh \left( \frac{2z - h}{z l} \right)}{\cosh \left( \frac{h}{2l} \right)} \right] \right)
\]

(8)

In the absorbent section, the couple stress fluid flow is governed by the modified Darcy’s law and represented as follows

\[q' = -\frac{k}{\mu(1-\beta)} V \rho'\]

(9)

Where \(q' = (u', v', w')\),

\(u', v', w'\) are the Darcy velocity components along \(x, y, z\) directions respectively.

Here in the porous region, the pressure is given by \(p^* \left( \frac{\eta}{\mu} \right) k\) where the permeability of the porous matrix is denoted by \(k\) and \(\mu\) is the isotropic viscosity of the fluid. If \(\frac{\eta}{\mu} = \sqrt{k}\), \(\beta = 1\) then in the lubricant the microstructure additive will block the pores in the porous matrix. If \(\beta < 1\), it percolate into the porous matrix. Hence the ratio of micro structure size to the pore size is represented as \(\beta\).

The pressure \(p^*\) satisfies the Laplace equations \(\nabla^2 p^* = 0\)
Effects of Frictional Force, Flow Rate and Shear Stress on Elliptic Plate using Couple Stress Fluid

\[ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0 \]  

(10)

At \( z=0 \), the B.C’s for velocity components are given by

\[ \frac{\alpha}{\sqrt{k}} (u - u') = \frac{\partial u}{\partial y} \]  

(11)

\[ \frac{\alpha}{\sqrt{k}} (v - v') = \frac{\partial v}{\partial y} \]  

(12)

\[ w = -w' \]  

(13)

\[ \frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 v}{\partial z^2} = 0 \]  

(14)

Equation (11) and (12) are B-J slip boundary conditions for the tangential velocity at the porous interface and \( \alpha \) is the slip coefficient.

Using the boundary conditions (11) and (12) in (3) and (4) we set the solutions given by

\[ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{12\mu}{f(h,c_1,c_2,l)} \left[ \frac{\partial h}{\partial t} + \frac{k}{\mu(1-\beta)} \frac{\partial p^*}{\partial z} \right] \]  

(15)

where

\[ f(h,c_1,c_2,l) = h^4(1+c_2) - 6h^2l c_1 \tanh \left( \frac{h}{2l} \right) - 12l^2 \left[ h - 2l \tanh \left( \frac{h}{2l} \right) \right] \]  

(16)

and

\[ c_1 = \frac{s}{h+s}, \]  

(17)

\[ c_2 = \frac{3}{1-\beta} \left[ \frac{2s^2 \alpha^2}{h^2 + sh} + \frac{s(1-\beta)}{h+s} \right] \]  

(18)

The relevant B.C is \( p^* = 0 \) on the boundary of the elliptic plate

\[ \frac{\partial p^*}{\partial z} = 0 \text{ at } z = -\delta \]  

(19)

Solving (10) using (19) we get

\[ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu}{f(h,c_1,c_2,l) + 128k / 1-\beta} \frac{d h}{d t} \]  

(20)

With \( p=0 \) on the boundary of the plates, the pressure \( p \) is obtained from (20)

\[ p = \frac{-12\mu}{f(h,c_1,c_2,l) + 128k / 1-\beta} \hat{p}(x,y) \]  

(21)

Where \( \hat{p}(x,y) \) denotes the Poisson equation

\[ \frac{\partial^2 \hat{p}}{\partial x^2} + \frac{\partial^2 \hat{p}}{\partial y^2} = -1 \]  

(22)

Since \( p=0 \) on the boundary \( \hat{p}(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}) \) which is zero on the boundary of the plate.

The shear stress along the surface is given by

\[ \tau = \frac{\partial u}{\mu} \]  

(23)

Using equation (7) in (22) we get

\[ \tau = \frac{\partial p}{\partial x} \left\{ (2z-h) - 2l \left[ \frac{\sinh \left( \frac{2z-h}{2l} \right)}{\cosh \left( \frac{h}{2l} \right)} \right] \right\} \]  

(24)

At \( z=h \), shear stress is given by
\[ \frac{\partial p}{\partial x} \left\{ h - 2l \tanh \left( \frac{h}{2l} \right) \right\} \]  
(24)

Similarly at the lower surface \( z = 0 \) we get
\[ \frac{\partial p}{\partial x} \left\{ -h + 2l \tanh \left( \frac{h}{2l} \right) \right\} \]  
(25)

Frictional force can be calculated as follows
\[ F = \int_0^l \tau \, dx \]  
(26)

Where \( \tau = \frac{\partial p}{\partial x} \left\{ -h + 2l \tanh \left( \frac{h}{2l} \right) \right\} \) for upper plate and

\[ P = \frac{-12 \mu \frac{dh}{dt}}{f(h, c_1, c_2, l) + 12 \beta k / (1 - \beta)} \left[ 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right] \lambda \] 

where \( f(h, c_1, c_2, l) = h^3 (1 + c_2) - 6h^3 l c_1 \tanh \left( \frac{h}{2l} \right) 12l^2 \left[ h - 2l \tanh \left( \frac{h}{2l} \right) \right] \)

Using these in (26) we get
\[ F = \frac{12 \lambda \mu \frac{dh}{dt}}{f(h, c_1, c_2, l) + 12 \beta k / (1 - \beta)} \left[ \frac{h}{2l} \tanh \left( \frac{h}{2l} \right) \right] \]  
(27)

The flow rate is given by
\[ Q = \int_0^k u \, dz \]  
(28)

Using equation (7) in (28) and integrating with respect to \( z \) we get
\[ Q = \frac{-\lambda \mu}{a^3} G(\mu, h, c_1, c_2, l, \beta) \left[ -\frac{h^3}{6} + 2l \left( h - \frac{1}{l} \tanh \left( \frac{h}{2l} \right) \right) \right] \]  
(29)

Where \( G(\mu, h, c_1, c_2, l, \beta) = \frac{12 \lambda \mu \frac{dh}{dt}}{f(h, c_1, c_2, l) + 12 \beta k / (1 - \beta)} \).

3. Results and Discussion

In the present article, the effects of frictional force, flow rate and shear stress of elliptic plates with couple stress fluid is presented. For numerical calculations of the results, the parameter value of \( s = 0.1, \alpha = 0.001, \beta = 0.2, \delta = 0.001, \kappa = 0.001 \) and \( \lambda = 0.1 \) are used.

Figures (2)-(4) shows the variation of shear stress \( \tau \) for the upper plate with the film thickness \( h \) for distinct values of couple stress parameter \( l \), the aspect ratio \( \lambda = b/a \) and isotropic viscosity \( \mu \) respectively. It is noted that the shear stress \( \tau \) decreases for rising values of couple stress parameter \( l \), \( \tau \) increases for rising values aspect ratio \( \lambda \) and for rising values of \( \mu \) the shear stress increases.

In figures (5)-(7) shows the variation of shear stress \( \tau \) for lower plate with film thickness \( h \) for distinct values of \( l, \lambda \) and \( \mu \) respectively. It is noted that the shear stress decreases for rising values of couple stress parameter \( l \), shear stress \( \tau \) increases for rising values of aspect ratio \( \lambda \) and viscosity coefficient \( \mu \).
In figures (8)-(10) shows the variation of frictional force $F$ with film thickness $h$ for various values of $l, \lambda$ and $\mu$ respectively and it is noted that the frictional force $F$ decreases for increasing values of $l$, increases for aspect ratio $\lambda$ and viscosity coefficient $\mu$.

In figures (11)-(12) shows the variation of flow rate $Q$ with film thickness $h$ for a range of values of $l$ and $\mu$ respectively and it is noted that the flow rate decreases for rising values of $l$ and increases for rising values of aspect ratio $\lambda$.

**Figure 2:** Variation of shear stress $\tau$ for upper plate with $h$, the film thickness for distinct values of $l$ with $s=0.1, \alpha=.001, \beta=0.2, \delta=0.001, \kappa=0.001, \lambda=0.1$.

**Figure 3:** Variation of shear stress $\tau$ for upper plate with film thickness $h$ for distinct values of $\lambda$. With $s=0.1, \alpha=.001, \beta=0.2, \delta=0.001, \kappa=0.001, l=0.1$. 
Figure 4: Variation of shear stress $\tau$ for upper plate with $h$, the film thickness for distinct values of $\mu$ with $s = 0.1, \alpha = .001, \beta = 0.2, \delta = 0.001, \kappa = 0.001, \lambda = 0.1$

Figure 5: Variation of shear stress $\tau$ for lower plate with film thickness $h$ for distinct values of $l$ with $s = 0.1, \alpha = .001, \beta = 0.2, \delta = 0.001, \kappa = 0.001, \lambda = 0.1$

Figure 6: Variation of shear stress $\tau$ for lower plate with film thickness $h$ for distinct values of $\lambda$ with $s = 0.1, \alpha = .001, \beta = 0.2, \delta = 0.001, \kappa = 0.001, l = 0.1$
**Figure 7:** Variation of shear stress $\tau$ for lower plate with film thickness $h$ for distinct values of $\mu$ with $s = 0.1, \alpha = .001, \beta = 0.2, \delta = 0.001, \kappa = 0.001, \lambda = 0.1$

**Figure 8:** Variation of $F$, the Frictional force with film thickness $h$ for distinct values of $l$ with $s = 0.1, \alpha = .001, \beta = 0.2, \delta = 0.001, \kappa = 0.001, \lambda = 0.1$

**Figure 9:** Variation of $F$, the Frictional force with film thickness $h$ for distinct values of $\lambda$ with $s = 0.1, \alpha = .001, \beta = 0.2, \delta = 0.001, \kappa = 0.001, l = 0.1$
**Figure 10:** Variation of $F$, the Frictional force with film thickness $h$ for distinct values of $\mu$ with $s = 0.1, \alpha = 0.001, \beta = 0.2, \delta = 0.001, \kappa = 0.001, \lambda = 0.1, \lambda = 0.1$.

![Graph showing Variation of $F$ with $h$ for distinct values of $\mu$.](image)

**Figure 11:** Variation of Flow rate $Q$ with film thickness $h$ for distinct values of $l$ with $s = 0.1, \alpha = 0.001, \beta = 0.2, \delta = 0.001, \kappa = 0.001, \lambda = 0.1$.

![Graph showing Variation of $Q$ with $h$ for distinct values of $l$.](image)

**Figure 12:** Variation of Flow rate $Q$ with film thickness $h$ for distinct values of $\lambda$ with $s = 0.1, \alpha = 0.001, \beta = 0.2, \delta = 0.001, \kappa = 0.001, \lambda = 0.1$.

![Graph showing Variation of $Q$ with $h$ for distinct values of $\lambda$.](image)
Conclusion
The following conclusion can be drawn based on the results discussed above:

- The shear stress decreases for the increasing values of couple stress parameter $l$ and increases with rising values of aspect ratio $\lambda$ and viscosity coefficient $\mu$ for both upper plate and lower plate.
- The frictional force $F$ decreases with the rising values of couple stress parameter $l$ whereas it increases with increasing values of aspect ratio $\lambda$ and viscosity coefficient $\mu$.
- The flow rate $Q$ increases with the diminishing values of couple stress parameter $l$ and rising values of aspect ratio $\lambda$.

References